

## TOK Knowledge Framework – Mathematics

### Methodology

Use of pure reason (is there any other?) from axioms to produce proofs of mathematical theorems.

A statement in maths is proved true only if it is proved.

Ironically, as maths is a way of describing the world, it does not rely on sense perception, only reason.

Mathematicians need intuition and imagination to prove theorems

### Concepts and Language

Uses a precisely defined set of symbols standing for abstract things like sets and relations.

Key terms such as axiom, deduction rule, conjecture, theorem, proof, logic, symmetry.

## Mathematics

### Historical Development

Why couldn't the Romans invent zero?  
Why could other cultures invent zero?  
Developments such as negative and irrational numbers have changed the way we view the world.  
Numbers and geometry important to historical developments in the arts – music, painting.  
Archaic number systems.

### Knowledge Questions

Does beauty and elegance equal “truth”?  
Is the language of mathematics universal?  
What place does emotion have in mathematics?  
Is mathematics an invention or a discovery?  
If maths is an abstract intellectual game, then why is it (sometimes) so good at describing the world?  
If maths exists in the real world, where is it exactly?  
If maths is an invention, why do we feel mathematical truths are purely objective facts about the world rather than a human construct?  
Structure order anarchy?  
Is there a structure to the universe?

### Scope and Application

Mathematics is concerned with quantity, shape, space and change – these are difficult to define.

We use maths as a tool to create models in the Natural and Human Sciences.

These models obviously have strengths and weaknesses – we have to assess how good the models are.

The possibility of a mathematical treatment is treated by some as a sign of rigour and certainty. How true is this? Think about Economics or Psychology.

We are often struck by beauty and elegance in mathematics, which we usually relate to the arts.

Maths seems to be universal and not tied to one culture

Mathematical truths have the appearance of being certain and timeless.

The language of mathematics – it's conventions, symbols and areas of study.

### Links to personal knowledge

Mathematical ability is often taken as a proxy for intelligence and ability to reason.

Major contributors to mathematics by people who cannot always explain their insights, often ascribing them to intuition, imagination or emotion.

Only one answer – this is comfortable and reassuring.